

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS

Statistics 1

Tuesday 18 JANUARY 2005

Afternoon

1 hour 20 minutes

2613

Additional materials: Answer booklet Graph paper MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
- Answer all questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive no marks unless sufficient detail of the working is shown to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The total number of marks for this paper is 60.

1 Asha keeps hens so that she has a regular supply of fresh eggs. During March, the numbers of eggs she collected on each day were as follows.

| | 4 | 5 | 8 | 4 | 6 | 5 | 6 | 7 | 7 | 10 | 11 | 18 | 12 | 9 | 5 10 | 6 | |
|-------|--|-------|-------|------|-------|--------|-------|-------|---|----|----|-----|-----|---|---------|---|----------------|
| | 5 | 6 | 4 | 5 | 5 | 6 | 7 | 8 | 8 | 13 | 10 | 11 | 14 | 9 | 10 | | |
| (i) | (i) Find the median number of eggs collected. | | | | | | | | | | | | [1] | | | | |
| (ii) | Find the upper quartile, lower quartile and interquartile range. | | | | | | | | | | | | [3] | | | | |
| (iii) | Draw | a box | x and | whis | ker p | lot fo | r the | data. | | | | | | | | | [3] |
| (iv) |) Using your answers to part (ii), identify any outliers. | | | | | | | | | | | [3] | | | | | |
| (v) | Calculate the mean number of eggs laid per day. | | | | | | | | | | | [2] | | | | | |
| (vi) | State suppo | | | | | | • | | - | | | • | | | - | | and (v) [3] |

2

2 In the sixth form at Eastport School, there are 45 students studying at least one of the three sciences, Biology, Chemistry and Physics.

Fig. 2 shows the number of students studying each science.

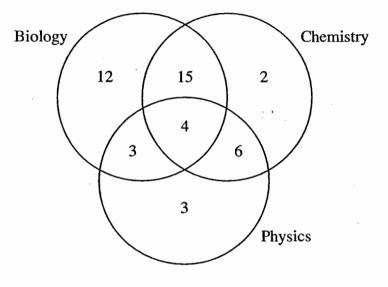


Fig. 2

- (i) Find the probability that one of these students, selected at random, is studying
 - (A) Physics,
 - (B) Chemistry,
 - (C) both Physics and Chemistry.
- (ii) Hence determine whether the events "studying Physics" and "studying Chemistry" are independent. [2]
- (iii) Find the probability that a student, selected at random, is studying Biology, given that the student is studying Chemistry. [2]
- (iv) Three students are selected at random. Find the probability that, between them, they are studying all three sciences although each is studying only one. [4]
- (v) Three students are selected at random. Find the probability that at least one of them is studying all three sciences.
 [3]

[3]

3 Canter is a company which has been asked to carry out a survey to discover how many adults watched a television programme called "Kendall's Way".

Canter decides to interview 1000 people by telephone.

(i) Why is this sample unlikely to be representative?

Canter has a list of the telephone numbers and ages of 5000 adults. From this list, the sample will be selected.

- (ii) Four of the interviewers who work for Canter have the following ideas for selecting the sample. In each case, name the type of sampling involved.
 - (A) Anthony would use an ordinary die to select a number between 1 and 5 inclusive, ignoring any 6s thrown. He would then take every 5th number, starting with the number thrown. So, for example, if he threw a 3, he would select the 3rd, 8th, 13th, 18th, ... from the list.
 - (B) Bernice would use a random number generator to select 1000 different random numbers between 1 and 5000 inclusive, and use these numbers to select from the list.
 - (C) Carmen would select the first 200 people on the list who were aged 16–25. She would then repeat the process for each of the age groups 26–35, 36–45, 46–55, and over 55.
 - (D) Dennis would use the same age groups as Carmen, but would take a random sample from each age group. The number of people selected in each age group would be proportional to the number of people in that age group nationally.
 [4]

It is decided to use the method suggested by Dennis. The proportions in each age group and the results of the survey are given in the table.

| | Age group | | | | | | |
|---|-----------|-------|-------|-------|---------|--|--|
| | 16–25 | 26–35 | 36-45 | 46–55 | Over 55 | | |
| Proportion in age group nationally | 15% | 20% | 18% | 17% | 30% | | |
| Proportion of age group watching Kendall's Way as found in survey | 6% | 9% | 15% | 15% | 25% | | |

- (iii) Find the sample size for each of the age groups.
- (iv) The population consists of 30 million people. Estimate how many people watched Kendall's Way.
- (v) Estimate what proportion of the people who watched Kendall's Way are over 55 years old.[2]
- (vi) Dennis decides he would like to interview 5 of the people sampled who were aged 26–35 and who watched Kendall's Way. How many different selections of 5 people can he make? [3]

[2]

[1]

4 Sweets called "Scruffies" are sold in packets of 18.

Scruffies come in a variety of colours, and market research shows that red is the most popular. Scruffies are packed randomly, and on average 25% are red.

- (i) Find the probability that a packet contains no more than 6 red Scruffies. [2]
- (ii) Find the probability that a packet contains exactly 4 red Scruffies. [3]

Because of a fault in the packing machine, one packet in ten now contains 19 Scruffies instead of 18.

(iii) Find the new probability that a randomly selected packet contains exactly 4 red Scruffies.

[5]

In order to increase sales, the manufacturer claims to have increased the proportion of red Scruffies. Eric decides to test the manufacturer's claim by purchasing a packet of Scruffies. (Assume that all packets now contain 18 Scruffies.)

(iv) The packet contains 8 red Scruffies. Does this support the manufacturer's claim at the 5% significance level? Write down your hypotheses clearly and show your working in full. [6]

Mark Scheme

| (i) | The median = 7 | B1 |
|-------|--------------------------------------|--|
| (ii) | The lower quartile $= 5$ | B1 |
| | The upper quartile $= 10$ | B1 |
| | The inter-quartile range $= 5$ | B1 ft |
| (iii) | | |
| | 4 5 7 10 18 | B1 Median B1 Quartiles B1 Whiskers |
| (iv) | Outliers at top $> 10 + 1.5(5)$ | M1 |
| | > 17.5 so 18 an outlier | A1 |
| | Outliers at bottom $< 5 - 1.5(5)$ | |
| | So no outliers at bottom | B1 |
| (v) | Mean $=\frac{244}{31}$ | M1 |
| | = 7.87 | A1 |
| (vi) | The graph exhibits positive skewness | B1 |
| | The mean > median | E1 |
| | UQ – median > median - LQ | E1 |
| | | 15 |

| (i) | A Prob (physics) $=\frac{16}{45}$ | B1 |
|-------|--|-------------------|
| | <i>B</i> Prob (chemistry) $=\frac{27}{45}=\frac{3}{5}$ | B1 |
| | <i>C</i> Prob (both) $=\frac{10}{45}=\frac{2}{9}$ | B1 |
| (;;) | If independent | |
| (ii) | If independent, P (phys and chem.) = P (phys). P (chem.) | M1 for product |
| | $\frac{16}{45} \times \frac{3}{5} \neq \frac{2}{9}$ | |
| | | |
| | Events are not independent | A1 |
| (iii) | Probability (Biology given Chemistry) | M1 for 19 in num. |
| | | |
| | $=\frac{19}{27}$ | |
| | 27 | M1 for 27 in den. |
| (iv) | Probability (3 diff single sciences) | M1 numerator |
| | 10 0 0 | M1 denominator |
| | $=6 \times \frac{12}{2} \times \frac{2}{2} \times \frac{3}{2}$ | |
| | $= 6 \times \frac{12}{45} \times \frac{2}{44} \times \frac{3}{43}$ | M1 for $\times 6$ |
| | = 0.005 (07) | A1 |
| (v) | Probability (at least one studies all 3) | M1 for 1- |
| | $=1 - \frac{41}{45} \times \frac{40}{44} \times \frac{39}{43}$ | M1 triple product |
| | =0.249 | A1 |
| | | 14 |
| | | |

| (i) | For example, one of | |
|-------|---|---------------------------|
| | Not everyone owns a telephone | E1 |
| | Not everyone will be available to answer | |
| (ii) | A – Systematic | B1 |
| (11) | B – Random | B1 B1 |
| | C – Quota | B1 |
| | D – Stratified (with proportional allocation) | B1 |
| (iii) | 16 – 25 150 sampled | |
| | 26 – 35 200 sampled | |
| | 36 – 45 180 sampled | |
| | 46 – 55 170 sampled | |
| | >55 300 sampled | M1 A1 |
| (iv) | (.06)(15)+(.09)(20)+(.15)(18)+(.15)(17)+(.25)(30) | M1 sum of pairs |
| | = 15.45% of 30 million | M1 multiply by 30 million |
| | = 4,635,000 | A1 |
| (v) | Proportion $=\frac{(.25)(30)}{15.45}$ | M1 for division |
| | = 0.485 | A1 |
| (vi) | Number of $26 - 35$ year olds = (.09)(200) | M1 for product |
| | = 18 people | M1 for $\binom{n}{5}$ |
| | Number of selections $= \begin{pmatrix} 18\\5 \end{pmatrix} = 8568$ | A1 |
| | | 15 |

| (i) | $P(X \le 6) = 0.861 = 0.861$ | M1 for $(X \le 6)$ |
|-------|--|---|
| | | A1 |
| (ii) | X ~ B(18,0.25) | |
| | $P(X = 4) = {\binom{18}{4}} (0.25)^4 (0.75)^{14}$ | M1 for $\begin{pmatrix} 18\\4 \end{pmatrix}$ |
| | = 0.213 | M1 for (0.25) ⁴ (0.75) ¹⁴ A1 |
| | or $0.5187 - 0.3057 = 0.213$ tables | M1 for correct table M1 for correct subtraction A1 |
| (iii) | P (X = 4) = $(.9)(.213) + (.1)(.202)$ | M1 for 0.9, 0.1 |
| | = 0.212 | M1 for 0.213 M1 A1 for 0.202 A1 |
| (iv) | Let p be the probability that a Scruffie is red | |
| | $H_0: p = 0.25$ | B1 |
| | H ₁ : p > 0.25 | B1 |
| | $P(X \ge 8) = 1 - P(X \le 7)$ = 1 - 0.9341 | M1 for $1 - P(X \le 7)$ |
| | = 0.0569 > 5% | A1 for 0.0569 M1 for comparison |
| | Cannot reject H _O | |
| | No evidence of an increased proportion of red Scruffies | E1 for context |
| | | 16 |

Examiner's Report